

1. Prove that $f: \mathbb{N} \rightarrow \mathbb{N}$, defined by $f(m) = m^2 + m + 1$ for all $m \in \mathbb{N}$, is one-one but not onto.
2. Solve for x

$$\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$$
3. Prove that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$
4. If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$
 find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$.
- 5) If $f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x}, & x < \frac{\pi}{2} \\ a, & x = \frac{\pi}{2} \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2}, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ find the values of a and b
- 6) If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ find $\frac{dy}{dx}$
- 7) Examine the following function for continuity and differentiability at $x=1$ and $x=3$
 $f(x) = |x-1| + |x-3|$.

10) $\int \frac{dx}{\operatorname{cosec} x + \cos x}$

11) Evaluate by using properties of definite integrals.

(a) $\int_0^{\pi/2} \frac{x \sin x \cos x dx}{\sin^4 x + \cos^4 x}$ (b) $\int_0^1 \frac{\log(1+x) dx}{1+x^2}$

12) Using Integration find the area of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

13) Using integration find the area of the triangle ABC whose vertices are A(4,1) B(6,6) and C(8,4).

14) Solve the differential equation $(1+e^{2x}) dy + (1+y^2) e^x dx = 0$, given that $y=1, x=0$

15) Solve the differential equation $(x-y) \frac{dy}{dx} = x+2y$.

16) Solve $(1+y^2) dx + (x - e^{-\tan^{-1} y}) dy = 0$ given that $y=0, x=0$.

17) Find the image of the point P(2, -1, 5) in the line $\vec{r} = 11\hat{i} - 2\hat{j} - 8\hat{k} + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$

18) Find the equation of the plane through the line of intersection of the planes $3x - 4y + 5z = 10$ and $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

~ x ~