SBOA school & Jr. college

HOLIDAY ASSIGNMENT

Sub: Mathematics XI- standard

Evaluate the following limits:

1) $\lim_{x \to 3} \dfrac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$

2) $\lim_{x \to 2} \dfrac{\sqrt{1+4x} - \sqrt{5+2x}}{x-2}$

3) $\lim_{x \to 4} \dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

4) $\lim_{x \to \frac{\pi}{4}} \dfrac{\sin x - \cos x}{x - \frac{\pi}{4}}$

5) $\lim_{x \to \frac{\pi}{2}} \dfrac{1-\sin x}{\left(\frac{\pi}{2} - x\right)^2}$

6) $\lim_{x \to 0} \dfrac{\cot 2x - \operatorname{cosec} 2x}{x}$

7) $\lim_{x \to 0} \dfrac{3\sin x - \sin 3x}{x^3}$

8) $\lim_{x \to 0} \dfrac{\tan 2x - \sin 2x}{x^3}$.

9) Find the value of k, if $\lim_{x \to 1} \dfrac{x^4-1}{x-1} = \lim_{x \to k} \dfrac{x^3-k^3}{x^2-k^2}$.

Differentiate the following using first principle:

1) $y = \sin\sqrt{x}$

2) $y = \sqrt{\cos x}$

3) $y = \sin^2 x$

4) $y = \dfrac{1}{\sqrt{x+2}}$

5) $y = x \cdot \sin x$

6) $y = \dfrac{\cos x}{x}$

Differentiate the following:

1) $y = \dfrac{\sin x - x\cos x}{x\sin x + \cos x}$

2) $y = \dfrac{x\tan x}{\sec x + \tan x}$

3) $y = (x\sec x + x\cdot\operatorname{cosec} x)(x\cdot\tan x + x\cdot\cot x)$

4) $y = (x\sin x + \cos x)(x\cdot\tan x + \sec x)$

5) $y = \dfrac{x^5 - \cos x}{x\cdot\tan x}$

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1. Prove that $f: N \to N$, defined by $f(m) = m^2 + m + 1$ for all $m \in N$, is one-one but not onto.

2. Solve for x
$$\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$$

3. Prove that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

4. if $a \neq p$, $b \neq q$, $c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$.

5) If $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & x < \pi/2 \\ a & x = \pi/2 \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & x > \pi/2. \end{cases}$ is continuous at $x = \pi/2$ find the values of a and b

6) If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ Find $\frac{dy}{dx}$

7) Examine the foll. function for continuity and differentiability at $x = 1$ and $x = 3$
$$f(x) = |x-1| + |x-3|.$$

10) $\int \frac{dx}{\operatorname{cosec} x + \cos x}$

11) Evaluate by using properties of definite integrals.

(a) $\int_0^{\pi/2} \frac{x \sin x \cos x \, dx}{\sin^4 x + \cos^4 x}$ (b) $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$.

12) Using Integration find the area of the region enclosed between the circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$.

13) Using integration find the area of the triangle ABC whose vertices are $A(4,1)$ $B(6,6)$ and $C(8,4)$.

14) Solve the differential equation
$(1+e^{2x})\,dy + (1+y^2)\,e^x\,dx = 0$, given that $y=1$, $x=0$

15) Solve. the differential equation
$(x-y)\frac{dy}{dx} = x+2y$.

16) Solve. $(1+y^2)\,dx + (x-e^{-\tan^{-1}y})\,dy = 0$
given that $y=0$, $x=0$,

17) Find the image of the point $P(2,-1,5)$ in the line $\vec{r} = 11\hat{i} - 2\hat{j} - 8\hat{k} + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$

18) Find the equation of the plane through the line of intersection of the planes. $3x-4y+5z=10$ and $2x+2y-3z=4$ and parallel to the line $x=2y=3z$.

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